

The Cosmological Energy Density of Neutrinos from Oscillation Measurements

Kevork Abazajian

Theoretical Division, MS B285, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

Abstract. The emerging structure of the neutrino mass matrix, when combined with the primordial element abundances, places the most stringent constraint on the flavor asymmetries in the cosmological neutrino background and therefore its energy density. I review the mechanism of synchronized neutrino oscillations in the an early universe with degenerate (asymmetric) neutrino and antineutrino densities and the implications of refined measurements of neutrino parameters.

INTRODUCTION

The dawn of the era of precision cosmology has come with the observations of anisotropies in the cosmic microwave background (CMB) with the Wilkinson Microwave Anisotropy Probe (WMAP) over the whole sky to better than fundamental uncertainty over a wide range in anisotropy scale [1]. Combined with the three-dimensional galaxy distribution of the Sloan Digital Sky Survey [2], a consistent picture has emerged for the standard concordance cosmology: a universe dominated by dark matter and dark energy with structure growing from nearly scale-invariant adiabatic Gaussian density perturbations. In the simplest models, WMAP and SDSS measure the cosmological matter density to nearly 10% [3].

Given the success of the standard concordance cosmology, it is tempting to assume that the density of all cosmological matter and radiation components of the universe are known to great precision. However, the neutrino density, often simply assumed to be fixed to its standard model value, is actually only known to factors of its own magnitude when using the WMAP data alone [4].

One can hope to do better with primordial nucleosynthesis. During primordial nucleosynthesis, the nucleon beta-equilibrium weak interaction rates are sensitive to the electron neutrino and antineutrino densities. The cosmic expansion rate depends on the overall neutrino density, which sets when nuclear reactions freeze-out. These two effects can compensate each other and can produce primordial element abundances for deuterium, helium and lithium that are consistent with their observed abundances, as long as the nucleon density is increased to allow the nuclear rates to keep up with the required increased expansion rate [5]. The non-zero neutrino chemical potentials (or degeneracy parameters) of this model led to its description as degenerate big bang nucleosynthesis (DBBN). Since the nucleon (baryon) density is independently constrained by the CMB, the magnitude of deviations from non-zero neutrino chemical potentials was appreciably constrained from the original DBBN models, but still allowed neutrino densities over twice that of the standard value [6].

With the emergence of the mass and mixing spectrum of the active neutrino flavors, particularly in the large to maximal mixing angles of the solar and atmospheric neutrino oscillation solutions, it was proposed that the mixing could lead to the equilibration of neutrino asymmetries prior to nucleosynthesis in the studies of Refs. [7, 8].

The first attempt to solve the full evolution equations for the active neutrino system using was performed numerically by Dolgov, Hansen, Pastor, Petcov, Raffelt, and Semikoz [9], who found that the maximal mixing solution of the atmospheric results and large mixing angle solution of the solar neutrino problem invariably led to a near equalization of neutrino asymmetries between flavors. Therefore, DBBN, which required a large disparity between electron and muon or tau neutrino densities, would not be viable in a universe with the observed neutrino mass and mixing matrix. Analytic insight into the flavor asymmetries' equalization and a quantification of changes within the range of mixing parameters was studied by Wong [10], and Abazajian, Beacom and Bell [11]. The constraint imposed by the resulting equalizing transformations excludes DDBN and requires neutrino densities to be within $\sim 3\%$ of the standard value. Therefore, any non-standard cosmic radiation energy density must come from a more exotic phenomenon than photons and neutrinos .

SYNCHRONIZED OSCILLATIONS

In an elegant paper, Pastor, Raffelt & Semikoz [13] showed that the synchronization mechanism, initially studied in Refs. [12] can be framed in the representation of synchronized dipoles precessing in a magnetic field, with the orientation of the dipole representing the flavor content.

The system of mixed neutrinos in a dense, scattering, self-refractive environment must be handled in a density matrix formalism. The two-flavor neutrino density matrix is

$$\rho(p) = \begin{pmatrix} \rho_{\alpha\alpha} & \rho_{\alpha\beta} \\ \rho_{\beta\alpha} & \rho_{\beta\beta} \end{pmatrix} = \frac{1}{2} [P_0(p) + \boldsymbol{\sigma} \cdot \mathbf{P}(p)], \quad (1)$$

where $\mathbf{P}(p)$ as the neutrino's "polarization" vector, which can be represented as an individual "magnetic-dipole." The polarization vector describes asymmetries in flavor densities, such that $\mathbf{P}(p)^{initial} \propto [f_e(p, \xi_e) - f_\mu(p, \xi_\mu)]$, where $f_\alpha(p, \xi_\alpha)$ is the Fermi-Dirac distribution for a neutrino of flavor α with degeneracy parameter ξ .

The synchronized transformation can be described by the vector equations

$$\begin{aligned} \partial_t \mathbf{P}_p &= +\mathbf{A}_p \times \mathbf{P}_p + \alpha(\mathbf{J} - \bar{\mathbf{J}}) \times \mathbf{P}_p, \\ \partial_t \bar{\mathbf{P}}_p &= -\bar{\mathbf{A}}_p \times \bar{\mathbf{P}}_p + \alpha(\mathbf{J} - \bar{\mathbf{J}}) \times \bar{\mathbf{P}}_p, \end{aligned} \quad (2)$$

where neutrino scattering is negligible, \mathbf{J} denotes the individual neutrino polarizations integrated over momentum, over-bars refer to antineutrino quantities, and α is the strength of the neutrino self-potential: $\alpha(\mathbf{J} - \bar{\mathbf{J}}) \times \mathbf{P}_p$.

The general "magnetic field" vector \mathbf{A}_p includes terms incorporating vacuum mixing, a thermal potential from the charged-lepton background, and a potential due to asymmetries between the charged leptons, $\mathbf{A}_p = \vec{\Delta}_p + [V^T(p) + V^B] \hat{\mathbf{z}}$. Vacuum mixing is

incorporated by

$$\vec{\Delta}_p = (\delta m_0^2/2p)(\sin 2\theta_0 \hat{\mathbf{x}} - \cos 2\theta_0 \hat{\mathbf{z}}), \quad (3)$$

where $\delta m_0^2 = m_2^2 - m_1^2$ and θ_0 are the vacuum oscillation parameters.

The thermal potential V^T arises from the finite-temperature modification of the neutrino mass due to the presence of thermally populated charged leptons, and V^B is the background potential arising due to asymmetries in charged leptons. V^B is the crucial term in the case of the sun, but is negligible in the early universe.

If one ignores the non-linear neutrino self-potential, the evolution of the system is trivial: the “magnetic-field” vector points in the direction of the charged lepton thermal potential, in the $\pm \hat{\mathbf{z}}$ direction, which is also the initial direction of the polarization vectors in a flavor-asymmetric system, as in DBBN. The thermal potential initially dominates but decreases as the universe cools, eventually becoming comparable to $\vec{\Delta}_p$, the vacuum term. $\vec{\Delta}_p$ points in a direction determined by the vacuum mixing angle (Eq. 3), which for large mixing is close to the $\hat{\mathbf{x}}$ direction. Each neutrino polarization (the flavor descriptor) then follows its respective “magnetic-field,” whose final orientation is in the direction of $\vec{\Delta}_p$, and thus the cosmic flavor content, simply depends on the vacuum mixing angle.

When including the neutrino self-potential, the explicit solution can only be calculated numerically. Ref. [11] found that with the self-potential, the collective system behaves on average identically with the case when the self-potential is flatly ignored, even though the self-potential dominates all other terms by five or more orders of magnitude. Refs. [10, 11] showed that under certain approximations, the effect of the neutrino self-potential is to force all neutrino polarizations to follow a specific synchronization momentum’s \mathbf{A}_p , whose value is $\frac{p_{\text{sync}}}{T} = \pi \sqrt{1 + \xi^2/2\pi^2} \simeq \pi$, which is coincidentally very close to the average momentum of the Fermi-Dirac distribution $\langle p/T \rangle \simeq 3.15$. Of course, this is what the system average would follow without self-potential.

This remarkable coincidence allows for a dramatic simplification of the apparently initially intractable nonlinear evolution equations and allows a straightforward visualization of the general behavior of the neutrino gas for a variety of cases and mixing parameters. As described above, the transformation that leads to total or partial flavor equalization occurs at a temperature where the vacuum term and thermal potential are comparable. Since the vacuum term $\vec{\Delta}$ is proportional to δm^2 , larger δm^2 leads to transformations at higher temperature. And, since the final orientation of the flavor polarization vectors is in the $\vec{\Delta}$ direction, the level of total or partial flavor equalization is determined by the vacuum mixing angle [11].

OSCILLATION PARAMETERS AND THE EARLY UNIVERSE

The consequences of the emerging neutrino mass matrix structure for a universe that contains neutrino degeneracies is quite rich. The implications for each of the mass scales in a three-neutrino mixing frame-work and their mixings is as follows:

Atmospheric Neutrinos, δm_{23}^2 and θ_{23} : for the range of δm^2 preferred by the oscillation solution to the atmospheric neutrino results by Super-Kamiokande [14], flavor equilibration occurs at a temperature $T \sim 12 \text{ MeV}$ due to the presence of equilibrating

scatterings, and maximal mixing produces absolute equalization of flavor density asymmetries. If precision measurements of θ_{23} reveal a non-maximal angle, the equalization of neutrino density would be very close though not necessarily perfect, and an explicit calculation would be necessary since scattering is not negligible at $T \sim 12 \text{ MeV}$.

Solar Neutrinos, δm_{12}^2 and θ_{12} : δm^2 for the large mixing angle solution to the solar neutrino problem is much smaller than that of the atmospheric scale, so that the thermal potential dominates until a lower temperature. The transformation in this case occurs at $T \sim 2 \text{ MeV}$, sufficiently before the start of nucleosynthesis at $T \sim 1 \text{ MeV}$, disallowing DBBN. The level of equalization is dependent on the orientation of $\vec{\Delta}$, i.e., how “large” the large mixing angle is. Precise measurements of θ_{12} would determine the final vacuum vector orientation, what neutrino asymmetries can be accommodated by primordial nucleosynthesis [11], and therefore the maximum allowed cosmic neutrino density.

Neutrino Factories, Reactors and Long-Baseline Experiments, θ_{13} : a non-zero value θ_{13} close to the current upper limit can lead to equalization at higher temperatures than that from the solar scale [9]. Also, for an inverted neutrino mass hierarchy, a very small but non-zero θ_{13} can lead to a resonance at $T \sim 5 \text{ MeV}$ that would also enhance equalization [11]. An appreciable θ_{13} or inverted hierarchy would further tighten the limits on the maximum cosmic neutrino density.

In summary, the intertwining of cosmic neutrino scattering, decoupling, weak beta-equilibrium freeze-out, and primordial nucleosynthesis with the mass and mixing scales for neutrino transformations in degenerate cosmic neutrino scenarios is exciting, particularly since the mass scales could have placed the transformations much higher or lower than the primordial nucleosynthesis scale. Therefore, the exact nature of the neutrino mass and mixing matrix, especially if it contains further surprises, will illuminate exactly what cosmic neutrino scenarios are plausible.

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